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Review

Reviewed Work(s): Iteration of Rational Functions: Complex Analytic Dynamical Systems  
by Alan F. Beardon

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and, it is now generally agreed, is heavily dependent on the ability to access a well-structured knowledge base rapidly and efficiently. Experts use their extensive, well-structured knowledge to facilitate rapid, relatively effortless problem solving. (This view is so widely shared that the authors of the chapter on writing expertise wonder why so many accomplished writers report that writing never comes easily. But I digress.) The development of problem-solving expertise in a domain usually involves the acquisition of facts and procedures dedicated specifically to solving problems in that domain. General strategies may facilitate the use of facts and procedures, but they cannot replace them.

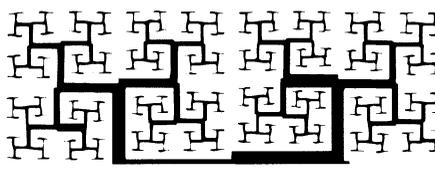
With slight variations, these propositions are restated in every chapter of this interesting book. Although there are multiple authors, they have a common perspective. As befits a volume emphasizing domain specificity, each chapter is devoted to a particular type of problem-solving skill. Problem-solving research has its roots in the study of recreational mathematics and strategic games, notably chess. This tradition is carried out in a chapter by the editors, although the message of the other chapters, echoed by Hunt in a concluding tour de force, is that game playing should no longer play such a central role, given the critical role of domain-specific knowledge, precisely because the context of a board game is so different from other domains.

This message might discourage both scientists and educators, for it clearly asks us to de-emphasize general processes (what some have called meta-cognitive strategies) and to look again at how specific knowledge is structured, stored in long-term memory and retrieved for use in particular situations. On the other hand, taken as a whole, the chapters emphasize the roles of practice, feedback and student activity in the acquisition of skill in solving a variety of problems. Finally, the book raises some questions: If problem solving is so heavily context-dependent, what will generalize from one situation to the next? How limited is transfer? How can we design instruction so as to maximize the probability of transfer to relatively novel situations? What is the role of meta-strategies and general processes?

The book is especially well suited for graduate courses in cognitive psychology, learning and instruction, or applied psychology. Computer enthusiasts will probably enjoy the chapters on computer simulations of complex environments, electronics troubleshooting and learning to use computers. Decision researchers will note the absence of subjective expected utility and decision trees from the cognitive models preferred by these researchers, and will think (as at least three

chapters invite us to do) about the links between problem solving and decision making. Scientists and professionals in fields not covered in this book will be interested in seeing how current research in other fields bears on their own concerns as experts and as educators. Readers expert in the fields represented will find the chapters to be useful summaries of the state of the art.—Arthur S. Elstein, *Medical Education, University of Illinois at Chicago*

## Mathematics and Computer Sciences



**Iteration of Rational Functions: Complex Analytic Dynamical Systems.** Alan F. Beardon. 280 pp. Springer-Verlag, 1991. \$39.95.

Most of us recognize the ubiquitous and beautiful computer-generated images of Julia sets; their fractal fingers and dizzying spirals appear on book covers, posters and T-shirts. The infinitely complicated bug-like image of the Mandelbrot set is familiar too. Few understand exactly what these images represent. Among other things they represent 14 years of progress in the melding of the fields of dynamical systems and analytic functions of a single complex variable. These two classical areas of mathematics were brought under a spotlight around 1979 with the advent of fast, inexpensive computers that can produce these spectacular sets in a few minutes, using public-domain software.

The subject of *Iteration of Rational Functions*, by Alan Beardon, has its roots in calculus. The most widely used iterative method for estimating the real roots of polynomials goes back to Newton. When, in 1879, Arthur Cayley tried to extend the method to approximate the complex roots as well, he noticed that successive iterations of Newton's algorithm sometimes led nearby points to completely different roots, especially if the points did not lie close to a root. Between 1918 and 1920 several papers by Gaston Julia and Pierre Fatou appeared on iterations of rational maps, offering a partial explanation for the occasional chaotic phenomenon observed by Cayley.

Suppose  $p(z) = p_0 + p_1z + p_2z^2 + \dots + p_nz^n$  is a polynomial of degree  $n$  whose roots are sought. The variable  $z$  represents a complex number, that is, an expression of the form  $z = x + iy$ , where  $x$  and  $y$  are real numbers (such as  $-2$ ,  $\pi$  or  $\sqrt{7}$ ) and  $i$  satisfies  $i^2 = i \times i = -1$ . The coefficients  $p_0, p_1, \dots,$

$p_n$  are complex numbers with  $p_n \neq 0$ . A root of  $p(z)$  is any complex value  $z$  that satisfies  $p(z) = 0$ . Starting from an arbitrary "guess,"  $z_0$ , Newton's method for estimating the zero closest to  $z_0$  involves iterating at  $z_0$  the rational map  $N(z) = z - p(z)/p'(z) = zp'(z)/p'(z)$  where  $p'(z)$  denotes the derivative of the polynomial  $p(z)$  and is defined exactly as for real variables.

The papers by Fatou and Julia studied iterations of a general rational map of the form  $R(z) = p(z)/q(z)$ , with  $p$  and  $q$  polynomials of degree  $n$  and  $m$  respectively, with no common factors. They showed that the complex plane is divided by  $R$  into two disjoint sets: a closed set  $J_R$  (later called the Julia set) on which the dynamics seem to be chaotic in the sense that nearby points exhibit very different long-term behavior under successive iterations of  $R$ , and an open set  $F_R$  (recently named the Fatou set) on which the set of mappings  $R^n = R \circ R \circ \dots \circ R$  (composition  $n$  times) is a family of maps that are continuous at each point  $z$  uniformly with respect to the integer  $n$ . Cayley's unpredictable points were lying in the Julia set for the map  $N$  given above.

Beardon's book offers a theoretical and rigorous treatment of the main results on the topology of Julia and Fatou sets for rational maps  $R$  and their influence on the dynamics of  $R$ . This is a mathematics text; the only computer images that appear are borrowed from other published and referenced sources. The first chapter consists of easy but illustrative examples, such as a comparison of the predictable dynamics of Möbius maps  $R(z) = (az + b)/(cz + d)$ ,  $ad - bc \neq 0$ , with the chaotic behavior of  $R(z) = z^2$ . The final chapter is also exclusively devoted to examples, illuminating the theory developed in the intervening chapters.

Starting from the level of a first-year graduate course in topology and complex analysis, the book is a self-contained study of the subject. The reader is guided through proofs that the Julia set is always nonempty and has positive Hausdorff dimension (provided the degree of  $R$  is at least 2), two results that guarantee some nontrivial computer output for interested hackers. The complete classification of the Fatou sets is given, including Dennis Sullivan's important "No Wandering Domains Theorem" published in 1985. The connections between periodic and critical points, and the nature of the dynamics, are carefully developed as well. If there is any disappointing omission of material it might be with regard to the Mandelbrot set.

After the work of Fatou and Julia, the field was essentially dormant until a computer-generated picture by Robert Brooks and Peter Matelski appeared in 1978 showing the unusual beetle-like shape of the set of complex parameters  $c$  for which the polynomial map  $R_c(z) = z^2 + c$  has a connected Julia set. Immediately after that, and independently, Benoit Mandel-

## Publishers' Addresses

brot produced a much more elaborate image of the same set at IBM, and the Mandelbrot set was born. Beardon gives Adrien Douady and John H. Hubbard's 1983 proof that the Mandelbrot set is connected, although none of the interesting connections between the intricate shape of the Mandelbrot set near a particular parameter and the corresponding Julia set there are explored. References to the theorems on this central topic are missing too.

Computer images continue to suggest theorems in this field. Anyone who would like to understand the subject on a serious mathematical level would benefit from reading this carefully written and well-referenced book on rational mappings. It is a good book for one's math library, and definitely not one for the coffee table.—*Jane Hawkins, Mathematics, University of North Carolina at Chapel Hill*

**The Little Book of Big Primes.** Paulo Ribenboim. xvii + 237 pp. Springer-Verlag, 1991. \$29.50.

Everyone has at one time or another taken an interest, however fleeting, in prime numbers. Paulo Ribenboim, however, is clearly a "prime nut," and this excellent, good-humored book is written for other (actually or potentially) incurable aficionados.

In the breeziest and most straightforward way, the author takes us from "Which is the oddest prime? It is 2, because it is the only even prime," to elaborate discussions of Carmichael numbers, the zeros of Riemann's zeta function, Sophie Germain primes and the Goldbach conjecture. Complicated definitions and theorems are explained, but long technical proofs are omitted, to keep the pace of the exposition from slackening and to limit the overall length of the book. Despite the almost flippant tone, the book pulls no punches and makes no hand-waving simplifications—it is a masterly presentation of hard mathematics and tough computation, for people who appreciate the difficulty of such things and the merit of those who succeed in obtaining the results. Let no one underestimate the difficulty of proving the primality of  $391581 \times 2^{216193} - 1$  (the largest known prime, as of this book's completion)!

Under six categories, the author presents a brief history of what is known and what is conjectured about prime (and associated) numbers. At every step, the largest or most complicated known results are listed. Indeed, the introduction presents this book as a compact number-theoretic addendum to the famous *Guinness Book of World Records*. For every record, there is a thorough explanation of the problem solved and a full confession of unproven conjectures and plain old ignorance, called "open problems." This genially reader-friendly tour de force, by a scientist with an ency-

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