

Errata and corrigenda: Ergodic and chaotic properties of Lipschitz maps on smooth surfaces

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ABSTRACT. Errata and corrigenda are given for *Ergodic and chaotic properties of Lipschitz maps on smooth surfaces*, New York J. Math. 18 (2012), 95–120.

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1. Erratum

There is an error in the remarks that occur between Equations (4.2) and (4.3) in [1]. Checking Equation (3.6) for the map g^* is necessary but not sufficient for (4.2) to hold as is implied by the remarks. The sentence

Therefore $G(x) = \varphi_{i+1} \circ g^* \circ \varphi_i^{-1}(x) = \varphi_i \circ g^* \circ \varphi_{i-1}^{-1}(x)$ is well-defined for every point $x \in X$.

does not hold. In fact G is not always well-defined when (3.6) holds. We remove the specific form of f from (3.6), and then we add condition (3.7); all equalities hold (mod 1):

$$(3.6) \quad g^*({\theta}) = -g^*({-\theta}),$$

$$(3.7) \quad \theta \in [0, 1/2] \Rightarrow g^*({\theta}) \in [0, 1/2].$$

We note that (3.6) and (3.7) in turn imply (3.8) and (3.9):

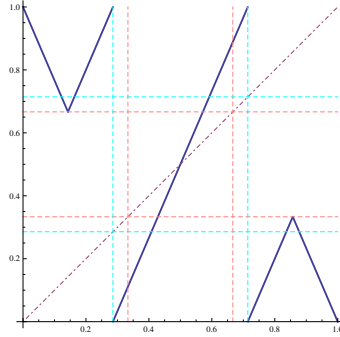
$$(3.8) \quad \theta \in (1/2, 1) \Rightarrow g^*({\theta}) \in (1/2, 1),$$

$$(3.9) \quad g^*({1/2}) = 0 \quad \text{or} \quad g^*({1/2}) = 1/2.$$

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FIGURE 1. The graph of g_s from Remark 5.2

Conditions (3.6) and (3.7) are sufficient for (4.2) to hold. Only (3.6) appears in [1]; this omission led to errors in the statements of Theorem 4.1 and 6.3, for which we offer corrections in the next section. The particular map g^* defined just before (4.1) does not satisfy (3.7) and in fact G is not well-defined in Theorem 4.1; Theorem 6.3 contains a similar error. While Theorem 6.3 has a simple correction using an alternative map discussed in [1], we can only prove a modified version of Theorem 4.1. A similar modification to Theorem 4.2 is also required.

2. Corrigenda

We start with the simple correction needed in Section 6.

2.1. A correction for Theorem 6.3. If we use the map described in Remark 5.2 instead of the map f_s used (i.e., use the degree one circle map whose graph is the reflection about $x = 1/2$ of the map f_s used), then Theorem 5.3 remains unchanged, and in Theorem 6.3, statement (2) is replaced by “(2) The points A and B are repelling fixed points of G_s .” The graph of the degree one map is shown in Figure 1.

2.2. Section 4 corrections. We cannot recover the statements of Theorems 4.1 and 4.2 as given in [1]. However we state revised versions here. Instead of $f_d(x) = dx \pmod{1}$ given in [1] (also written as $f(z) = z^d$), we use closely related maps denoted F_d , each one a d -to-one map with the property $|F'_d(x)| = d$ and which maps $[0, 1/2]$ onto $[0, 1/2]$. The formula for F_2 is:

$$F_2(x) = \begin{cases} 2x & \text{if } x \in [0, \frac{1}{4}), \\ 1 - 2x & \text{if } x \in [\frac{1}{4}, \frac{1}{2}), \\ 2 - 2x & \text{if } x \in [\frac{1}{2}, \frac{3}{4}), \\ 2x - 1 & \text{if } x \in [\frac{3}{4}, 1). \end{cases}$$

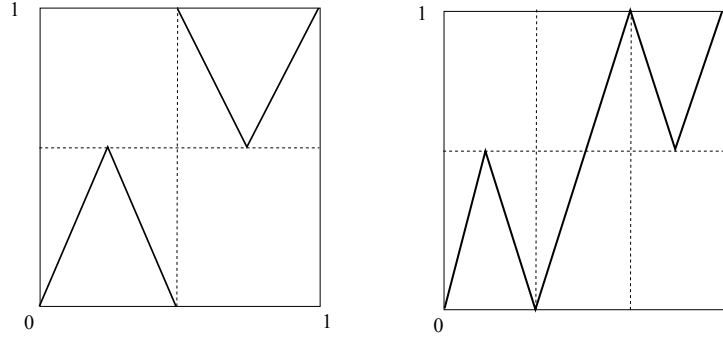


FIGURE 2. The graphs of F_2 and F_3

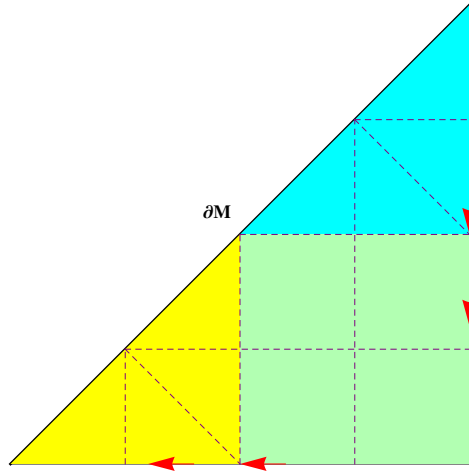


FIGURE 3. The ergodic decomposition and Bernoulli partitions of F_2^{*2} on \mathbb{M}

There is an analogous formula for each $d \geq 3$, and we describe the graph here. For $x \in [0, \frac{1}{2}]$, reflect any segment of the graph of $f_d(x) = dx \pmod{1}$ which lies above the $y = 1/2$ line across that line, and for $x \in (\frac{1}{2}, 1]$, reflect any segment below the line $y = 1/2$ across that line. The graphs for $d = 2$ and $d = 3$ are illustrated in Figure 2. Evidently F_d satisfies the properties in Equations (3.6)–(3.9), making G well-defined on nP .

The map F_d has two ergodic components (the intervals $[0, \frac{1}{2})$ and $[\frac{1}{2}, 1)$), and the map F_d^{*2} on the symmetric product $I^{*2} \cong \mathbb{M}$ (the Mobius band) has three ergodic components as shown in Figure 3; each color represents an ergodic component and a generating Bernoulli partition is shown within each ergodic component.

The revised Theorems 4.1 and 4.2 are as follows; the proofs are as in [1], but instead we use the maps F_d given here and make obvious minor modifications.

Theorem 4.1 (Revised). *Given any nonorientable compact surface X of genus $n \geq 2$, there exists a map $G : X \rightarrow X$ which is locally Lipschitz on X (Lipschitz in each coordinate chart), continuous, and smooth except on a finite number of curves, and satisfying:*

- (i) G preserves a smooth probability measure m_n on X .
- (ii) G has three ergodic components with respect to m_n .
- (iii) The restriction of G to each ergodic component is isomorphic to an n -point extension of a one-sided Bernoulli shift.
- (iv) On each ergodic component G is transitive and chaotic, but not topologically exact.
- (v) $h_{\text{top}}(G) = 2 \log d$.

Theorem 4.2 (Revised). *Suppose $(\mathbb{S}^1, \mathcal{B}, m, f)$ is any nonsingular d -to-one dynamical system satisfying the following conditions:*

- (1) f is continuous on \mathbb{S}^1 and differentiable except at finitely many points.
- (2) f is topologically exact.
- (3) f is weak mixing.
- (4) In additive coordinates, $f(1-x) = 1-f(x)$ for all $x \in [0, 1]$ and $f([0, 1/2]) = [0, 1/2]$.

Then for any nonorientable compact surface X of genus > 1 , f defines a d^2 -to-one nonsingular map G on X with respect to a smooth measure μ , has at most three ergodic and chaotic components and G is continuous and differentiable μ -a.e.

At the end of Section 4 in [1] we show how the measure theoretic entropy of the examples we construct can be reduced; this still holds with a slight variation on the examples T_p given. By reflecting the inner two line segments of the graph shown in Figure 8 of [1] about the line $y = 1/2$, we can still obtain G_p of arbitrarily small entropy as claimed.

References

- [1] GOODMAN, SUE; HAWKINS, JANE. [Ergodic and chaotic properties of Lipschitz maps on smooth surfaces](#). *New York J. Math.* **18** (2012), 95–120. Zbl 06032704.

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